

PRIFYSGOL CYMRU : UNIVERSITY OF WALES

**DEGREE EXAMINATIONS 2011
ARHOLIADAU GRADD 2011**

**Swansea Metropolitan University
Prifysgol Fetropolitan Abertawe**

Faculty of Applied Design and Engineering

NAME OF COURSE/PROGRAMME: *BEng Motorcycle Engineering*

YEAR or LEVEL (state which): *Year III*

NAME OF MODULE/UNIT: *Advanced Motorcycle Dynamics*

NAME OF EXAM PAPER:
(if different to module name)

DATE OF EXAMINATION:

EXAMINATION TIMES:

Start Time:
Finish Time:
Time Allowed:

INSTRUCTIONS TO CANDIDATES:

For all questions, take $\rho_{air} = 1.167 \text{ kg m}^{-3}$

An equation sheet is provided with this exam paper

Answer 3 from 4 Questions

FACULTY REF:

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Question 1 – (In-Plane Dynamics)

The motorcycle in its plane of symmetry can be represented as three rigid bodies whose vibrating motion is described by four independent coordinates:

- The vertical displacement of the sprung mass centre.
- The pitching rotation of the sprung mass.
- The vertical displacements of the *two* unsprung masses.

Draw a schematic representation of the system.

(6 marks)

Show that the equations of free, undamped motion can be described as:

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_{yg} & 0 & 0 \\ 0 & 0 & m_f & 0 \\ 0 & 0 & 0 & m_r \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\mu} \\ \ddot{z}_f \\ \ddot{z}_r \end{bmatrix} + \begin{bmatrix} k_f + k_r & (p-b)k_f - bk_f & -k_f & -k_r \\ (p-b)k_f - bk_r & (p-b)^2 k_f + b^2 k_r & bk_r & -(p-b)k_f \\ -k_f & bk_r & k_r + k_{p_r} & 0 \\ -k_r & -(p-b)k_f & 0 & k_f + k_{p_f} \end{bmatrix} \begin{bmatrix} z \\ \mu \\ z_f \\ z_r \end{bmatrix} = 0$$

(14 marks)

Question 2 – (Steering)

Consider a motorcycle with a wheelbase, $p = 1.4\text{m}$, characterized by the following tyre properties:

$$k_{\lambda_f} = 12\text{rad}^{-1}$$

$$k_{\lambda_r} = 14\text{rad}^{-1}$$

$$k_{\varphi_f} = 0.9\text{rad}^{-1}$$

$$k_{\varphi_r} = 0.85\text{rad}^{-1}$$

Determine the steering ratio, ξ , and describe the behavior of the motorcycle when the steering ratio has this value.

(4 marks)

Determine the ‘critical velocity’.

(4 marks)

How does this ‘critical velocity’ effect the steering behavior of the motorcycle, and what measures could be taken to avoid any possible loss of control in this type of situation.

(6 marks)

Describe the principle formulation of the multi-body model for the motorcycle in steady turning. Your description should encompass the number of rigid bodies, degrees of freedom, the number of equations that derive from the equilibrium conditions and what physical aspects the equations represent.

(6 marks)

Question 3 – (Stability)

The motorcycle has two main types of vibrational modes:

The in-plane and out-of-plane modes of vibration for a motorcycle are affected by different parameters.

- a. Explain what influences the in-plane vibrational modes and what effect this has on the motorcycle and the rider.

(3 marks)

- b. Explain what parameters influence the out-of-plane modes, and what mathematically determines the natural frequency of the oscillating modes.

(4 marks)

Experimentally, the motorcycle can exhibit a number of different oscillatory and non-oscillatory modes. Define the following terms and describe how they manifest themselves in motorcycle dynamics and how they are modeled mathematically.

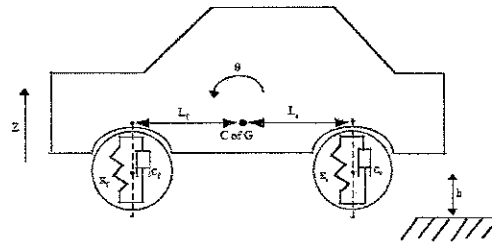
- c. Capsize
- d. Wobble
- e. Weave

(7 marks)

- f. Sketch the root-locus plot for a motorcycle cornering at a constant speed, showing where the different oscillatory modes lie in both the real and imaginary planes. Show also the areas of stability and instability.

(6 marks)

Question 4 – (In Plane Dynamics)



The figure above represents the characteristics of a 'half car' or basic motorcycle model. The front and rear suspension systems are modelled as spring-damper systems and the contribution of the tyre characteristics is neglected.

The vehicle has both pitch and bounce degrees of freedom. They are represented in the model by four states: vertical displacement, vertical velocity, pitch angular displacement, and pitch angular velocity.

Derive equations for:

- The front suspensions influence on bounce.
- The pitch contribution of the front suspension.
- The forces for the body motion.
- The moments for the body motion.

(10 marks)

The tyre model proposed by Pacejka in 1991 is still very much in use today. The approach is substantially empirical and the results reproduce the real behaviour of the tyre very well. The entire model revolves around what is called a 'magic formula', which is a single expression that can be used to represent the longitudinal driving or braking force, the lateral force or the moment around the z-axis. The expression is:

$$Y(x) = y(x) + S_v$$

$$y(x) = D \sin\{C \tan^{-1}[Bx - E(Bx - \tan^{-1} Bx)]\}$$

$$X = x + S_h$$

Where B, C, D and E are four parameters, S_v indicates the translation of the curve along the y-axis and S_h indicates the translation of the curve along the x-axis.

e. Sketch the curve of the Pacejka "Magic Formula", accurately labelling the parameters outlined in the expression above.

f. Explain the physical significance of each of the parameters in the Pacejka expression.

(10 marks)



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Programme:	BEng Motorcycle Engineering	Year:	Level 6
Module/Unit:	Advanced Motorcycle Dynamics		
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Question No	Question	Marks
1a	The four coordinate system to represent the 3 body system can be described as	6
b	firstly resolving forces, for undamped motion: we get $M\ddot{z} + (k_f + k_r)z + (p-b)k_s\theta - bk_r\theta - k_f z_s - k_r z_r = 0$ or $M\ddot{z} + (k_f + k_r)z + ((p-b)k_s - bk_r)\theta - k_f z_s - k_r z_r = 0$	

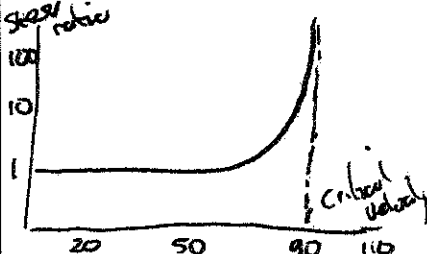
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Question No	Question	Marks
14	Taking moments, using rotational coordinates we have $I\ddot{\theta} + (P-b)k_f z - b k_r z + (P-b)^2 k_f \theta + b^2 k_r \theta - (P-b)k_f z_f + b k_r z_r = 0$ giving: $I\ddot{\theta} + ((P-b)k_f - b k_r)z + ((P-b)^2 k_f + b^2 k_r)\theta - (P-b)k_f z_f + b k_r z_r = 0$ Now, resolve forces for M_f: $M_f \ddot{z}_f - k_f z - (P-b)k_f \theta + (k_f + k_{t_f})z_f = 0$ and $M_r \ddot{z}_r - k_r z + b k_r \theta + (k_r + k_{t_r})z_r = 0$ These 4 equations give rise to the Matrix $\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & M_f & 0 \\ 0 & 0 & 0 & M_r \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \\ \ddot{z}_f \\ \ddot{z}_r \end{bmatrix} = \begin{bmatrix} k_f + k_r & (P-b)k_f - b k_r & -k_f & -k_r \\ (P-b)k_f - b k_r & (P-b)^2 k_f + b^2 k_r & -(P-b)k_f & b k_r \\ -k_f & -(P-b)k_f & k_f + k_{t_f} & 0 \\ -k_r & b k_r & 0 & k_r + k_{t_r} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ z_f \\ z_r \end{bmatrix} = 0$ The natural frequencies are determined by determining the related eigenvalue problem...	14
Question Total		20

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Question No	Question	Marks
2a	The steering ratio: for front $\frac{1 - K_{\phi r}}{K_{\lambda r}} = \frac{1 - 0.85}{14} = 0.01$ for rear $\frac{1 - K_{\phi f}}{K_{\lambda f}} = \frac{1 - 0.9}{12} = 0.0083$ $\frac{r}{l} = 1 + \frac{0.01 - 0.0083}{1} = 1.0017 \therefore > 1$ Also eff. $\frac{r}{l} = \frac{\lambda_r}{\lambda_f} = \frac{0.01}{0.0083} = 1.2$ This clearly shows an oversteer situation 6/ $V_c = \sqrt{\frac{GP}{\frac{1 - K_{\phi r}}{K_{\lambda r}} - \frac{1 - K_{\phi f}}{K_{\lambda f}}}} = \sqrt{\frac{1.4 \times 9.81}{(0.01 - 0.0083)}}$ $V_c = 89.83 \text{ ms}^{-1}$ As we approach this speed, we are entering an unstable steer state, where the oversteer becomes so large as to be broadly unstable. This case of 'rear slide' can however be controlled by the use of gyro steer (countersteering) to maintain a consistent sliding situation, as in speedway or motorcycling. This is however NOT easy 	(4) (4) (6)

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Question No	Question	Marks
	A motorcycle can be described as a system of six rigid bodies: Sprung steering components, un-sprung steering components, rear frame (including frame, engine, tank and rider), rear swinging arm and the two wheels. The rider is considered to be a rigid body firmly attached to the frame. The speed of travel V is the speed of the contact point of the rear wheel. When no slip is present, V is directly proportional to the angular velocity of the rear wheel and is directed along the wheel symmetry plane. The distributed aerodynamic forces which air exerts on the motorcycle are taken into account by considering drag, lift and lateral forces acting at the center of mass of the rear frame (F_D, F_L, F_S) and three aerodynamic torques (M_{Ax}, M_{Ay}, M_{Az}) The interaction between each tire and the road is represented by three forces (vertical load, longitudinal and lateral forces) acting at the geometric contact point and by three torques (overturning, rolling resistance and yaw torque), acting along the three independent axes. The tire forces and torques are non-linearly dependent on the roll angle and slip quantities. The equilibrium conditions give six equations: Three force equilibrium equations; Three moment equilibrium equations: equilibrium around the X -axis (roll), the Y -axis (pitch) and the Z -axis (yaw). In addition we have two equations that give the lateral forces as functions of side-slip and camber angles. Once the roll angle θ and the steering angle δ are assigned, the eight equations allow us to obtain the eight unknowns: Forward velocity V ; Vertical forces N_f and N_r, applied respectively to the front and rear wheels; Lateral forces F_{Sf} and F_{Sr} applied respectively to the front and rear wheels; Sideslip angles λ_f, λ_r The driving force, S Finally, the equilibrium of the front and/or the rear frame around the steering axis gives the torque exerted by the rider and applied to the handlebars, which provides and equal and opposite reaction on the rear frame.	

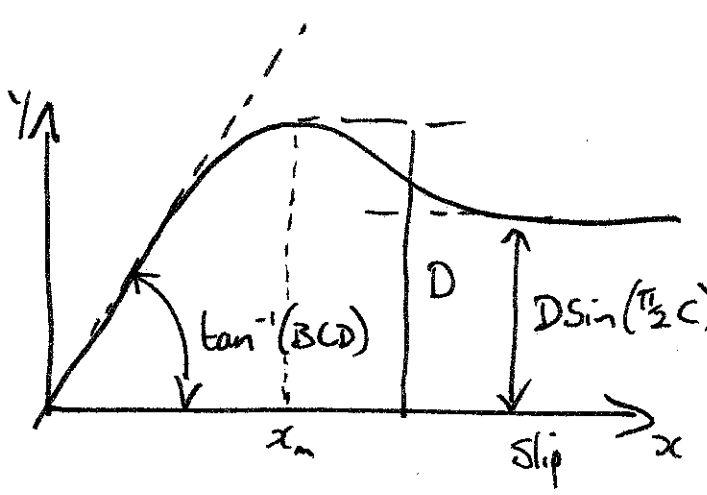
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Question No	Question	Marks
	The in-plane vibrational modes of the motorcycle are influenced by the frame, suspension and the wheel motions in the vertical plane. The in-plane modes are related to ride comfort and road-holding. The out-of-plane modes of vibration involve the roll, yaw, steering angle and steering head lateral displacement. The out-of-planes modes are generally related to the vehicle stability and handling. The natural frequencies of the oscillating modes (both in and out of plane) are given by the imaginary parts of the eigenvalues. The capsize motion is in essence a roll motion deeply influenced by the rider action on the handlebar. This motion is always unstable as the motorcycle can be thought of as an inverted pendulum. It is modelled mathematically as: $\varphi = \varphi_0 e^{\frac{t}{\tau}}$ Where tau is a positive time constant, showing capsize is always unstable. Wobble is an oscillation of the front assembly around the steering axis that can become unstable at fairly low to mid speeds. Wobble frequency goes up as the trail of the motorcycle increases and as the front-frame inertia decreases. Mathematically this is: $I_{Af} \ddot{\delta} = -c\dot{\delta} - F_{Sf}a_n + M_f g b_f \delta \sin \epsilon + N_f a_n \delta \sin \epsilon$ Weave is an oscillation of the whole motorcycle, but mainly an effect seen at the rear end of the motorcycle. It depends on the CofG of the motorcycle, the wheel inertia, the caster angle and trail and the cornering stiffness of the rear tyre. The weave mode is technically generated by the coalescence of two non-oscillating modes; body capsize and steering capsize. Mathematically it is the solution of: $I_{Ar} \ddot{\psi} = -c\dot{\psi} - F_{Sr}l$	

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Question No	Question	Marks
4/a	front influence on bounce :	
	$C_f \ddot{z}_f + C_f(P-b)\ddot{\theta} + K_f z + K_f(P-b)\theta$	(2)
b	front influence on pitch :	
	$C_f(P-b)\ddot{z} + C_f(P-b)^2\ddot{\theta} + K_f(P-b)z + K_f(P-b)^2\theta$	(2)
c	forces on body motion :	
	$M\ddot{z} + (C_f + C_r)\dot{z} + C_f(P-b)\dot{\theta} - C_r b\dot{\theta} + (K_f + K_r)z + (K_f(P-b) - K_r b)\theta = 0$	(3)
d	$I\ddot{\theta} + C_f(P-b)\dot{z} - C_r b\dot{z} + C_f(P-b)^2\dot{\theta} + C_r b^2\dot{\theta} + (K_f(P-b) - K_r b)z + (K_f(P-b)^2 + K_r b^2)\theta = 0$	(3)
e		

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Question No	Question	Marks
220/ 44/	for the Pacejka formula Parameter D represents the peak value (only with $E < 1$ and $C \geq 1$) and depends on the vertical load Parameter C controls the asymptotic value assumed by the curve as the slip tends to infinity, and in this way determines the resulting P_{max} of the curve. Parameter B determines the slope of the curve from the origin Parameter E characterises the curvature near the peak and at the same time determines the position of the peak itself The gradient at the origin is given by the product of BCD	⑩
Question total		20